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KINEMATICS OF FLOW OF A SLOW SUSPENDED-PARTICLE FLUX ABOUT A SPHERE AS APPLIED TO THE SEDIMENTATION OF A BIDISPERSE SUSPENSION

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A study is made of the kinematics of flow of fine particles about a coarse one in the process of sedimentation of the fine particles. An approximate analytical solution for the increase in the settling rate of fine particles, which is in good agreement with the approximation dependence obtained earlier as a result of numerical experiment, has been found. It has been shown that a relatively weak entrainment of most of the fine particles arriving at the peripheral region of the cell around the coarse particle is of primary importance for the effect of acceleration of sedimentation. A comparison with the available experimental data is made.

In a strongly rarefied suspension, particles of different diameters settle at different rates, which leads to the segregation of fractions by size. In this case, one can calculate the sedimentation rate for any fraction, assuming that an arbitrarily taken particle settles as a single one. As the volume concentration of the solid phase in the suspension increases, the motion of the particles is affected by neighboring particles and the sedimentation rate changes. It has been noted, for example, that in polydisperse and bidisperse suspensions, in the case of certain (not too high) concentrations, fine fractions sediment much more rapidly than single particles of the same size [1–5]. It can be assumed that this effect is responsible for the nonmonotonic dependence of the separation curve of a hydrocyclone, which indicates increased removal of a fine-grained fraction through an output device intended to remove the coarse-grained part of a material arriving at the apparatus [6, 7].

Purposeful use of the effect is possible, for example, in removing mechanical impurities from water, which is often carried out in large-scale units (precipitators) operating with the use of the process of sedimentation [8]. Here the sedimentation of the most slowly settling finest fractions could be accelerated by the addition of a relatively coarse rapidly sedimenting powder (for example, sand) of the appropriate coarseness rather than of special chemical preparations, i.e., coagulators.

In explaining and describing the phenomenon of acceleration of the sedimentation of a fine fraction, we will assume that it is caused by the entrainment of particles by the flow induced by a coarse particle.

Hydrodynamic Model. We restrict ourselves to the case of a bidisperse suspension. Since the particles of radius r_c of the coarse fraction settle more rapidly than fine ones of radius r_f in a coordinate system tied to a coarse particle, fine particles will move in the direction opposite to the direction of sedimentation (from the bottom upward). Let us consider the motion of fine particles in the vicinity of one of the coarse particles, which will be called spheres for brevity.

Since the volume of the boundary layer around each moving particle of radius r is in proportion to r^3, for particles that differ strongly in size we can consider that a fine particle, being in the vicinity of a coarse one (sphere), is completely "buried" in the boundary layer of the latter and does not affect the flow in this layer.

To describe the hydrodynamic field around a sphere in the case of slow flow about it ($\mathrm{Re}_c = u_c r_c/\nu = 2g(\rho_p - \rho_{liq})r_c^3/(9\rho_{liq}\nu^2) \le 1$, which is quite justified for particles with $r_c = 10\ \mu$m) we can use, in the first approximation, the Stokes solution [9]

$$u(x, y) = u_c\left(1 - \frac{1}{4r}\left(3 + \frac{1}{r^2}\right) + \frac{3x^2}{4r^3 r_c^2}\left(\frac{1}{r^2} - 1\right)\right), \tag{1}$$

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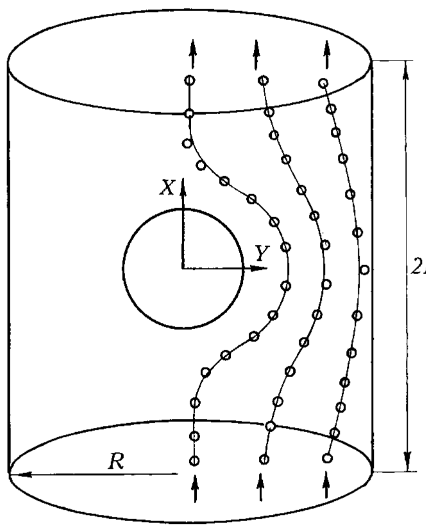


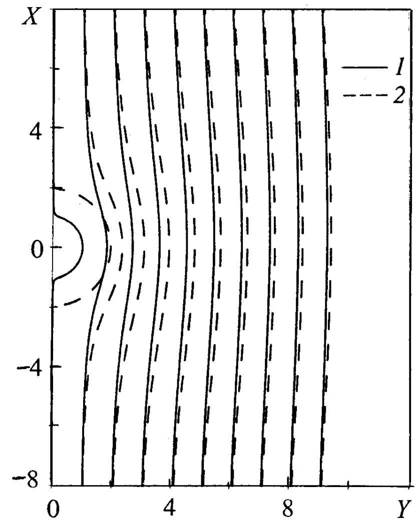


Fig. 1. Model of a computational cell.

Fig. 2. Paths of individual particles (D = 10): 1) U_f = 0 and 2) 0.3.

$$v\,(x, y) = u_c \frac{3xy}{4r^3 r_c^2}\left(\frac{1}{r^2} - 1\right), \tag{2}$$

where flow at a large distance from the sphere is assumed to be directed along the x axis; $r^2 = (x/r_c)^2 + (y/r_c)^2$, $(x/r_c)^2 \ge 1$ and $(y/r_c)^2 \ge 1$. Then the motion of each fine particle relative to the sphere can be calculated on the basis of the following system of kinematic equations:

$$\frac{dx}{dt} = u\,(x, y) - u_f\,, \;\; \frac{dy}{dt} = v\,(x, y)\,. \tag{3}$$

We select the radius of a coarse particle r_c as the length scale, the Stokes rate of settling of this particle $u_c = 2(\rho_p - \rho_{liq})r_c^2 g/(9\eta)$ as the velocity scale, and r_c/u_c as the characteristic time, i.e., we introduce the following dimensionless variables: time $\tau = tu_c/r_c$, coordinates $X = x/r_c$, $Y = y/r_c$, axial and radial velocities $U = u(x, y)/u_c$ and $W = v(x, y)/u_c$, and parameters of the Stokes rate of sedimentation of fine particles $U_f = u_f/u_c$.

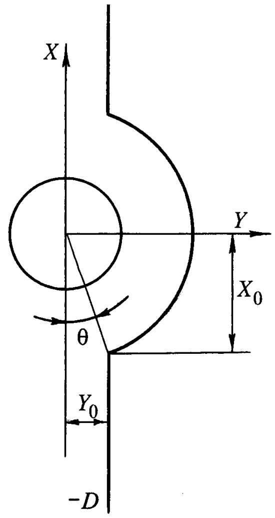
Such consideration holds for flow about a unit sphere. The application of the model to description of the sedimentation of an ensemble of particles is possible if the distance between the spheres R is considered to be larger than their size r_c.

Next we assume that the sphere is at the center of a cell which will be constructed in the form of a cylinder of radius R and height $2R$ (Fig. 1). We seek to find the characteristics of the flow of fine particles in such a cell. The parameters of the problem which occur because of the introduction of the cellular structure will be as follows: size of the cell $D = R/r_c$ and coordinate Y_0. It would appear natural that fine particles are distributed uniformly on the plane of entrance to the cell $x = -D$, i.e., the fraction of the particles inflowing thorough a circle of radius y is equal to y^2/R^2.

Unlike [4, 10, 11], where the outcomes of the model have been established based on numerical calculations, below the emphasis is on an approximate analytical examination of the properties of solutions.

Characteristics of Paths. The calculations of [10] give an idea of the paths of particles (Fig. 2). On the basis of Fig. 1, the entire trajectory of enveloping of the sphere by a fine particle can be subdivided into individual characteristic portions. This is particularly clearly demonstrated for low values of Y_0 (Fig. 3).

According to the scheme (Fig. 3), the particle, entering the cell, approaches the sphere, moving rectilinearly. At a certain point (X_0, Y_0), the particle shifts to a circular orbit of radius $r_0 = \sqrt{X_0^2 + Y_0^2}$. The value of X_0 can be determined by equating the velocities along the X and Y axes: $U(X_0, Y_0) - U_f = W(X_0, Y_0)$. Assuming



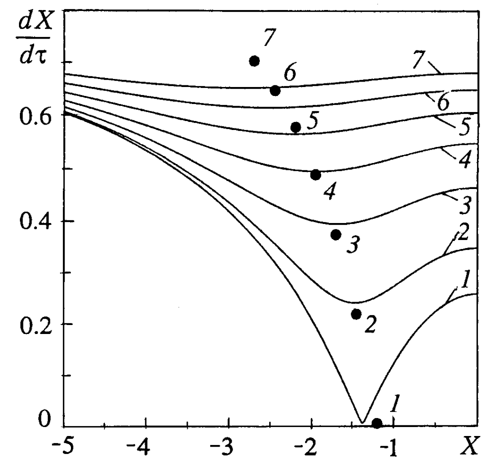


Fig. 3. Scheme of subdivision of the path of a particle into portions.

Fig. 4. Influence of the coordinate of the point of entry of a particle into the cell on the dependence of the longitudinal velocity of the particle on the coordinate X: 1) Y_0 = 0.01, 2) 0.5, 3) 1.0, 4) 1.5, 5) 2.0, 6) 2.5, and 7) 3.0.

Y_0 to be a small quantity and X_0 to be a quantity much more than unity and employing the expansions in the vicinity of the axial line $Y = 0$

$$U(X, Y) \approx 1 - U_f - \frac{3}{2r} + O\left(\frac{y^2}{x^2}\right) \approx 1 - U_f + \frac{3}{2X} + O\left(\frac{y^2}{x^2}\right), \tag{4}$$

$$W(X, Y) \approx \frac{3Y}{4X^2} + O\left(\frac{y^2}{x^2}\right), \tag{5}$$

we arrive at the equation

$$(1 - U_f)\,\alpha + \frac{3}{2X_0} = \frac{3Y_0}{4X_0^2}, \tag{6}$$

whence we find

$$X_0 = \frac{-3}{4(1 - U_f)\,\alpha}\left[1 + \sqrt{1 + \frac{4(1 - U_f)\,\alpha Y_0}{3}}\right]. \tag{7}$$

In actual fact, (7) is applicable at small Y_0; therefore, it is more convenient to employ the expansion

$$X_0 = \frac{-3}{2\alpha(1 - U_f)}\left[1 + \frac{\alpha(1 - U_f)\,Y_0}{3}\right]. \tag{8}$$

The correction factor α takes approximate account of the discarded terms of high orders in (4); $\alpha = 1.4$ has been introduced based on a comparison with numerical calculations. The particle velocity (longitudinal along the X axis) at the points of change of the character of motion is minimum and can be found by substitution of (8) into expression (4):

$$U(X_0, Y_0) \approx (1 - U_f)\,\alpha + \frac{3}{2X_0} = \alpha(1 - U_f)\left[\frac{Y_0\alpha(1 - U_f)}{3 + Y_0\alpha(1 - U_f)}\right]. \tag{9}$$

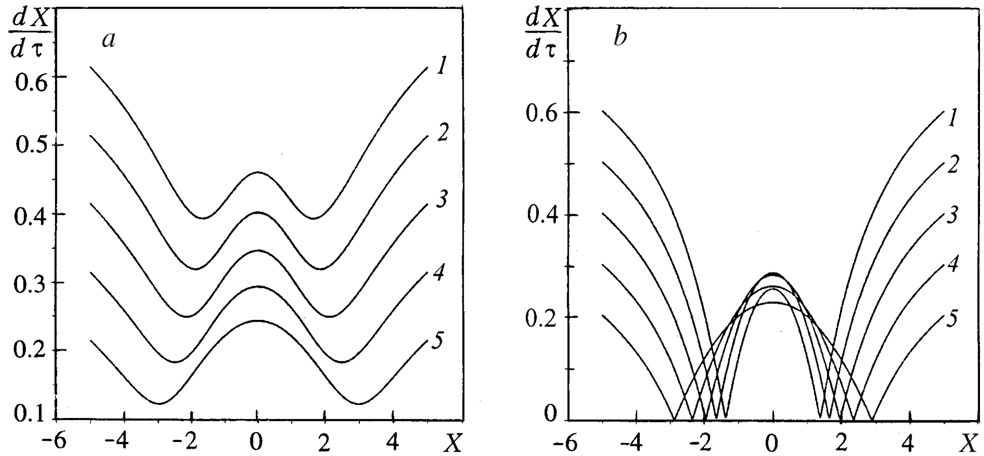


Fig. 5. Influence of the intrinsic rate of sedimentation of a particle on the profile of its longitudinal velocity for D = 5 and different values of the coordinate of the point of entry into the cell: a) Y_0 = 1 and b) Y_0 = 0.01: 1) U_f = 0.1, 2) 0.2, 3) 0.3, 4) 0.4, and 5) 0.5.

Dependences (8) and (9) (points 1–7 in Fig. 4) correspond approximately to the minima on the plots of the longitudinal velocity of a particle which entered the cell with the coordinate Y_0 as a function of the coordinate X.

The value of $U(X_0, Y_0)$ can be used as the velocity scale of a fine particle in traversing the cell. For small Y_0 this velocity is low (fine particles are entrained by coarse ones), while for large Y_0 it formally depends just on $1 - U_f$. From Fig. 4 it is seen that the particles entering the cell at a large distance from the central line $Y_0 = 0$ are not involved in circular motion and pass through the cell with a constant velocity $\propto 1 - U_f$. For equal diameters of the particles of both fractions $U_f = 1$ and $U(X_0, Y_0) = 0$; hence, there is no relative motion of the particles.

To describe motion in a circular orbit it is better to use the solution of Stokes equations in spherical coordinates [12]. For the tangential velocity we have

$$r_0 \frac{d\theta}{d\tau} = \left(1 - \frac{3}{4r_0} - \frac{1}{4r_0^3}\right) \sin\theta - U_f \sin\theta \,. \tag{10}$$

The last term of (10) is a projection of the intrinsic sedimentation rate of a fine particle onto the θ axis. By using the equality $dX = r_0 \sin\theta d\theta$, we find the velocity relation for $X = 0$ and $X = X_0$:

$$\frac{\dot{X}(\pi/2)}{\dot{X}(\theta_0)} = \frac{1}{\sin^2(\theta_0)} \approx \frac{1}{\theta_0^2} = \frac{X_0^2}{Y_0^2} \,. \tag{11}$$

When Y_0 is small this relation is large. It tends to 1 for particles entering the cell at a large distance from the central line, as is seen in Fig. 4.

The influence of the parameter U_f on the dependence of the longitudinal velocity for two values of the coordinate Y_0 is shown in Fig. 5. It is noteworthy that the coarser (more inert) the particle of the fine fraction, the weaker the differences in the values of its velocity during the traversal of the cell. Clearly, for particles entering the cell in the vicinity of the central line the influence of the flow generated by the sphere is stronger than for particles penetrating into the cell at the periphery.

Time Characteristics. The main features of motion of a particle with time in the case of flow about a sphere can be seen in Figs. 6 and 7. The motion is symmetric relative to the cross section $X = 0$ for all the initial coordinates. This, in particular, means that it will suffice to consider the motion until the equator is reached, after which the motion is repeated. At a large distance from the sphere, the motion is nearly uniform for particles with all the initial coordinates. The particles with high values of Y_0 also continue their uniform (and rectilinear) motion

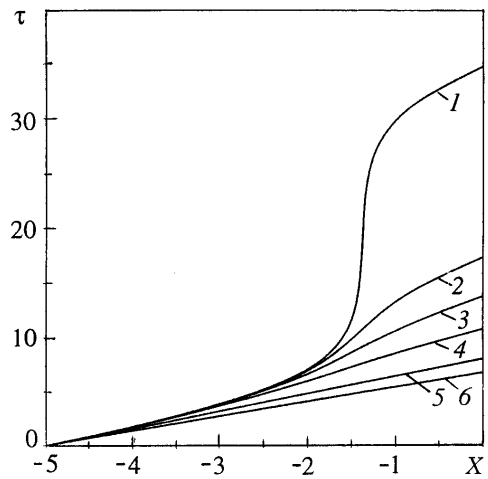


Fig. 6. Influence of the coordinate of the entry of a particle into the cell on the dependence of the running time of the particle on the coordinate X for U_f = 0.1 and D = 5: 1) Y_0 = 0.01, 2) 0.25, 3) 0.5, 4) 1.0, 5) 2.5, and 6) 5.0.

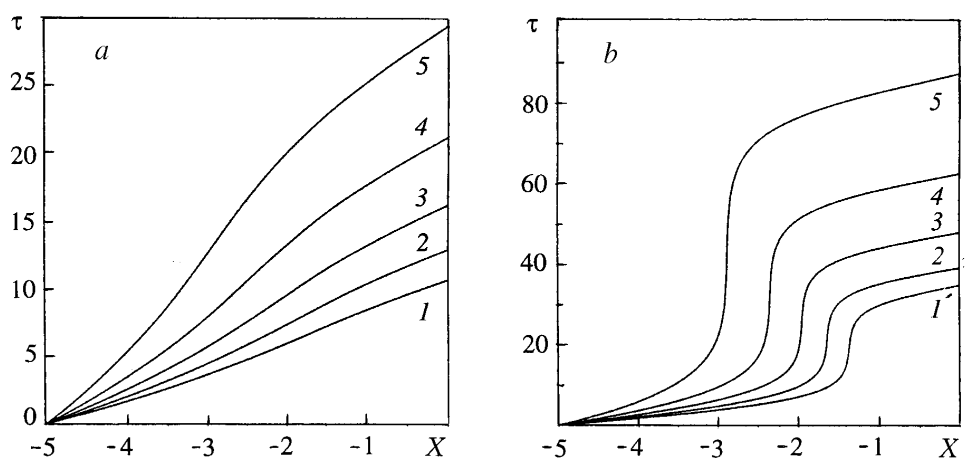


Fig. 7. Kinematics of motion of particles of different size for D = 5 and different values of the coordinate of the point of entry into the cell: a) Y_0 = 1 and b) Y_0 = 0.01: 1) U_f = 0.1, 2) 0.2, 3) 0.3, 4) 0.4, and 5) 0.5.

in traversing the layer $|X| \approx 2$, whereas the particles started for low values of Y_0 sharply slow down their motion in the axial direction, which is related to the shift to a circular orbit.

The motion of a particle along a straight line from $-D$ to X_0 approximately corresponds to the equation

$$\frac{dX}{d\tau} = 1 - U_f + \frac{3}{2X} + \dots , \tag{12}$$

which should be solved for $\tau = 0$ and $X = -D$.

Consideration can be simplified by taking a certain average velocity. Analyzing Fig. 6, we can see that on this portion the uniformity of the motion along the X axis with a velocity (based on the expansion (4)) approximately equal to

$$\langle U \rangle = 1 - U_f - \frac{3}{2D\sqrt{1 + (Y_0/D)^2}} \tag{13}$$

is sufficiently accurate. With such consideration, the time of traversal of the rectilinear portion will be

$$\tau_1 = \frac{D + X_0}{\langle U \rangle} . \tag{14}$$

In accordance with the note to formula (7) for large Y_0, the rectilinear portion of motion of the particle extends from $X = -D$ up to $X = 0$. In this case

$$\tau_1 \approx \frac{D}{1 - U_f - \frac{a}{D}} , \tag{15}$$

where $a = 3/(2\sqrt{1 + (Y_0/D)^2})$ can vary between 3/2 and $3/(2\sqrt{2})$.

Integration of the equation of motion in a circular orbit $r_0 = \sqrt{X_0^2 + Y_0^2}$ with the initial condition $\tau = 0$ and $\theta = \theta_0 = \arctan\ (Y_0/X_0)$ yields

$$\tau_2 = S\ (r_0) \int_{\theta_0}^{\pi/2} \frac{d\theta}{\sin\theta} = -\ S\ (r_0) \ln \tan \left(\frac{\theta_0}{2}\right) = S\ (r_0) \ln \left(\frac{2\ |X_0|}{Y_0}\right), \tag{16}$$

where

$$S\ (r_0) \approx r_0 \left[1 - \frac{3}{4r_0} - \frac{1}{4r_0^3} - U_f\right]^{-1} . \tag{17}$$

In subsequent evaluations, we can take $S(r_0) \approx 3/2$, assuming that $Y_0/X_0 << 1$ and $U_f << 1$.

When $Y_0 = 2|X_0|$, from (7) we formally have the equation

$$Y_0 = \frac{3}{2\ (1 - U_f)\ \alpha} \left[1 + \sqrt{1 + \frac{4\ (1 - U_f)\ \alpha Y_0}{3}}\right],$$

which leads to the evaluation of the disappearance of the circular portion of the path for

$$Y_* \approx \frac{6}{(1 - U_f)\ \alpha} . \tag{18}$$

This feature of motion is seen in Fig. 4. The maximum value for $|X_0|$ is accordingly equal to

$$|X_*| = \frac{3}{(1 - U_f)\ \alpha} . \tag{19}$$

Expressions (18) and (19) should be considered as estimating ones for the limit of the possibility of subdividing the total path into individual portions of rectilinearly uniform and circular motion. The assumption $Y_0/X_0 << 1$ taken in deriving (7) does not hold upon reaching this limit.

The total time of stay (residence) of the particle in the cell is represented in the form

$$\tau_r = 2\ (\tau_1 + \tau_2) . \tag{20}$$

Let us evaluate the contribution of each of these terms. When the values of Y_0 are low, $\tau_2 > \tau_1$ predominates; with distance from the central line the situation is the reverse. Beyond the circle of radius $Y_0 \approx 2|X_0|\ \exp\ (-\tau_1/S)$, the time of stay of the particle in the cell is determined mainly by the period τ_1. The fraction of particles which stay in the cell for a long time due to the strongly curved path is equal to

$$\frac{Y_1^2}{D^2} \approx \frac{9}{D^2} \exp\left(-\frac{2D}{3}\right) << 1 . \tag{21}$$

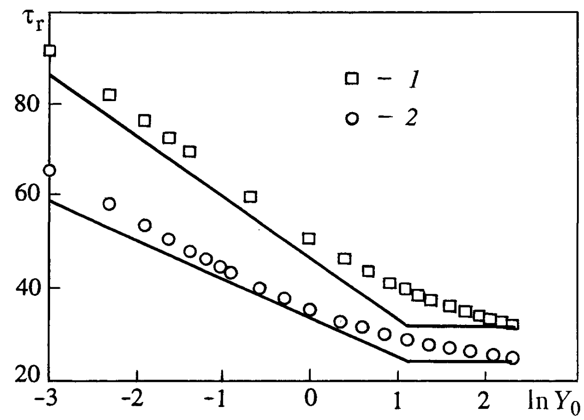


Fig. 8. Time of stay of a particle vs. coordinate Y_0 for D = 10: points, numerical calculation [11] [1) U_f = 0.1 and 2) 0.3]; curves, formula (20).

Accordingly, the role of such particles in the total flux of a sedimenting suspension is insignificant. The results of calculations from formula (20) are given in Fig. 8.

Sedimentation Rate. Since the axial velocity of the flux of a liquid and hence fine particles will be different at different points of the cell, the time of stay in the cell will also be different for each fine particle. If $t_j = t(y_j)$ is the time of traversal of the control volume by a particle having the coordinate y_j in the entrance cross section $x = -R$, the mean velocity of this particle relative to a coarse one will be determined as $u_p(y_j) = 2R/t(y_j)$ while the flux-mean velocity (under the assumption of a uniform distribution of particles in the entrance cross section of the cell) can be computed as

$$\langle u_p \rangle = \frac{1}{\pi R^2} \int_0^R 2\pi y_j u_p(y_j)\, dy_j = \frac{4}{R} \int_0^R \frac{y_j}{t(y_j)}\, dy_j \,. \tag{22}$$

The mean rate of sedimentation of fine particles in a laboratory coordinate system will be taken to be

$$u_s = u_c - \langle u_p \rangle \,. \tag{23}$$

Since there are two parameters in the problem, the solution sought for the sedimentation rate $U_s = u_s/u_c$ acquires the form

$$U_s = F(U_f, D) \,. \tag{24}$$

Equality (22) in dimensionless form will be

$$U_s = 1 - \frac{4}{D} \int_0^D \frac{Y}{\tau_r(Y)}\, dY \,. \tag{25}$$

The expression for τ_r is given by formula (20). In integrating, we have to subdivide the interval [0, D] into two portions. The formula for computations is transformed as

$$U_s = 1 - \frac{2}{D} \left[\int_0^{Y_*} \frac{Y}{\tau_1 + S(r_0) \ln\left(\dfrac{2\,|X_0|}{Y}\right)}\, dY + \int_{Y_*}^{D} \frac{Y}{\tau_1}\, dY \right] = 1 - \frac{2}{D} [J_1 + J_2] \,. \tag{26}$$

Here (if the weak dependence of S and τ_1 on Y is disregarded), the integral

$$J_1 = \int_0^{Y_*} \frac{YdY}{\tau_1 + S \ln\left(\frac{2\,|X_0|}{Y}\right)} = \frac{1}{S} \int_0^{Y_*} \frac{YdY}{\ln\left[\exp(\tau_1/S)\,\frac{2\,|X_0|}{Y}\right]}$$

can be expressed, using the new variable $z = 2\ln\left[\exp(\tau_1/S)\frac{2\,|X_0|}{Y}\right]$, in terms of the Airy function tabulated in [13]:

$$J_1 = \frac{4X_0^2 \exp(2\tau_1/S)}{S} \int_{2\tau_1/S}^{\infty} \frac{\exp(-z)\,dz}{z}\,.$$

Since we have the asymptotic expansion $\int_x^{\infty} \frac{\exp(-z)}{z}\,dz \approx \sum_{n=0}^{N} (-1)^n \frac{n!\,\exp(-x)}{x^{n+1}}$ for the Airy function at high values of the argument, restricting ourselves to two terms of the expansion ($N = 2$), we obtain (when $2\tau_1/S(r_0) >> 1$)

$$J_1 \approx \frac{2X_0^2}{\tau_1} - \frac{X_0^2 S}{\tau_1^2}\,. \tag{27}$$

Furthermore (if the weak dependence of τ_1 on Y_0 is disregarded), we have

$$J_2 = \frac{D^2 - 4X_0^2}{2\tau_1}\,. \tag{28}$$

With allowance for this,

$$U_s \approx 1 - \frac{2}{D}\left[\frac{2X_0^2}{\tau_1} - \frac{X_0^2 S}{\tau_1^2} + \frac{D^2 - 4X_0^2}{2\tau_1}\right] = 1 - \frac{D}{\tau_1}\left[1 - \frac{X_0^2 S}{D^2 \tau_1}\right] = 1 - \frac{D}{\tau_1} + \frac{X_0^2 S}{D^3}\left(\frac{D}{\tau_1}\right)^2. \tag{29}$$

The second term in (29) is small and must not be taken into account in what follows. In actual fact, this means that the contribution of the particles with a strongly curved path is disregarded. This circumstance is well reflected by inequality (21).

Let us evaluate D/τ_1 by eliminating particles with $Y_0 << 1$ and employing (15):

$$\frac{D}{\tau_1} \approx 1 - U_f - a/D\,. \tag{30}$$

Having taken a value of 1.3, which is intermediate between $3/(2\sqrt{2})$ and 3/2, to evaluate a and having substituted (30) into (29), we will have

$$U_s \approx U_f + 1.3\,\frac{1}{D}\,. \tag{31}$$

Expression (31) differs little from the formula derived in processing numerous computer-aided calculations (the constant 1.35 instead of 1.3 at the term $1/D$, which is quite explainable by the approximateness of the value taken for a).

With allowance for the relationship between $1.5D^3 = C_{V_c}^{-1}$ and $U_f = u_f/u_c = (d_f/d_c)^2$ relation (31) leads (in dimensional variables) to the equation

$$\frac{u_s}{u_f} = 1 + 1.48\, C_{V_c}^{1/3}\left(\frac{d_c}{d_f}\right)^2. \tag{32}$$

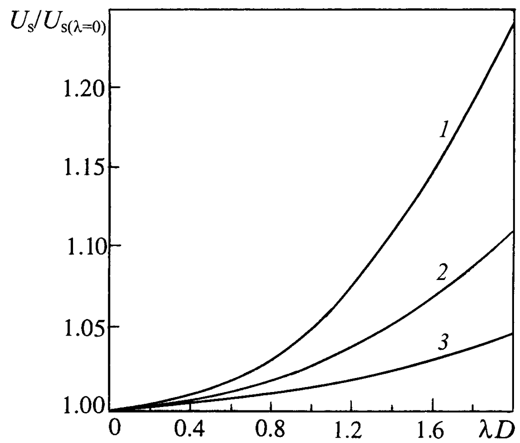


Fig. 9. Evaluation of the influence of neglect of the contribution of particles entering the cell in the vicinity of the axis of symmetry on the mean sedimentation rate: 1) D = 5, 2) 10, and 3) 20.

Thus, the dependence (found in the course of numerical investigations [10, 11]) of the increase in the sedimentation rate of the fine fraction of a bidisperse suspension on the concentration of coarse particles and on the ratio of the sizes of the particles of both fractions has been explained in the course of the approximate analysis.

Nontrivial, in particular, is the circumstance that a relatively weak entrainment of the main fraction of fine particles arriving at the cell around the sphere in its peripheral region and not the small fraction of fine particles whose paths are distorted in the course of flow about the sphere is of prime importance for the effect of acceleration of sedimentation.

This can be illustrated by direct calculations of U_s from formula (25), in which the value λD (where λ is a variable coefficient of "neglect" of the particles in the vicinity of the axial line) is taken as the lower limit of the integral. Figure 9 shows the calculated dependences of the ratio of the mean sedimentation rate of fine particles for $\lambda \neq 0$ to the sedimentation rate computed for $\lambda = 0$ on the quantity λD. The calculations have been carried out for three sizes of cells (three volume concentrations of the coarse fraction) for the relation of the diameters of the fine and coarse particles $U_f = 0.3$. It is seen that for D = 5, 10, and 20 neglect of the particles entering the cell in the vicinity of the axial line ($\lambda D < 1$) changes the value of the mean sedimentation rate by no more than 5%.

Comparison with Experiment. Experiments on flotation (the physics of the phenomenon is the same as for the case of sedimentation) of a polydisperse emulsion of particles (droplets) of paraffin oil in a vertical column have been described in [3]. Kumar et al. derive the relation which makes it possible to compute the rate of flotation (sedimentation) based on the measured values of the concentration for each fraction of the particles by size and of the total volume concentration of the disperse phase at two points of the column: at the site of supply of the emulsion and at a certain distance from it.

For the case of a 2.6% emulsion the data of [3] can be presented in the form of a linear dependence between the logarithms of the particle volumes and the corresponding flotation rates (the flotation rate was measured in centimeters per second while the particle size was measured in microns):

$$\log (u_s) = 0.34 \log \left(\frac{\pi}{6} d_f^3\right) - 1.79 \,. \tag{33}$$

Kumar et al. [3] also give the dependence for the Stokes rate, which is approximated for this case by the formula

$$\log (u_f) = 0.66 \log \left(\frac{\pi}{6} d_f^3\right) - 3.07 \,. \tag{34}$$

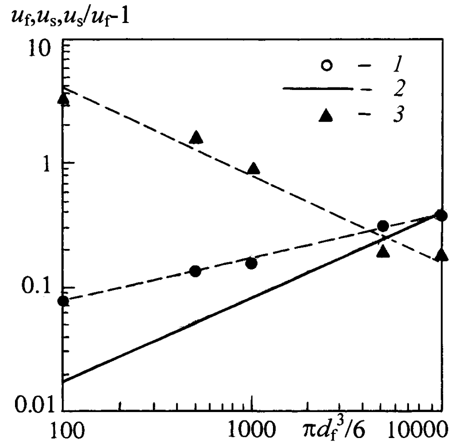


Fig. 10. Experimental data [3] on flotation (sedimentation) of a polydisperse suspension of particles of paraffin oil (concentration 2.6%) in water which confirm the effect of the entrainment of fine particles by coarse ones: 1) flotation rate; 2) rate according to the Stokes law; 3) ratio of 1 to 2.

These dependences and the dependence of the ratios of the measured flotation rates to the values of the Stokes rates which is obtained on their basis

$$\log\left(\frac{u_s}{u_f} - 1\right) = -0.69 \log\left(\frac{\pi}{6} d_f^3\right) + 1.96 \qquad (35)$$

are given in Fig. 10.

Agreement between the experimental data and the theoretical dependence of the sedimentation rate on the size of the particles of the fine fraction (32) is satisfactory. Clearly, a comparison of the theory constructed for a bidisperse suspension and the experiments carried out for a polydisperse emulsion is possible only on a limited scale.

If we take into account that the effect of entrainment is the stronger, the larger the ratio of the sizes of the coarse and fine particles and take, in (32), the size d_c equal to the maximum size of particles in the emulsion (in the case in question this yields approximately $d_c^3 = 1.9 \cdot 10^4\ \mu\text{m}^3$, as is seen in Fig. 10), we can reduce formula (35) to the form

$$\log\left(\frac{u_s}{u_f} - 1\right) = -0.79 + 1.03 \log\left(\frac{(1.9 \cdot 10^4)^{1/3}}{d_f}\right)^2 . \qquad (36)$$

Comparing (36) and (32) and having taken the total concentration of the dispersed material $C_V = 0.026$ instead of C_{V_c}, we find that the differences in these formulas are that the coefficients of log $(d_c/d_f)^2$ differ by approximately 3% and relation (36) written in the form (32) also yields that 0.55 will stand in (32) instead of the constant 1.48. Such a difference in the values of the constants is not surprising if we take into account the replacement of C_{V_c} by C_V. If, conversely, the constant 1.48 is considered to be correct, from the comparison of (32) and (36) we find that the concentration of coarse particles entraining fine ones is roughly equal to 0.0013, i.e., is about 5% of the total volume of the particles.

The influence of the concentration of particles on the effect of entrainment has not been studied in [3] systematically. However, it is of interest to note the measurements (given there) of the rates of flotation of the particles of the emulsion of paraffin oil in water with addition of salt (the concentration of NaCl was 0.01 mole/liter) for a concentration of the disperse phase of 0.7%. As the processing of the data shows, the flotation rates in this case obey the law

$$\log\left(\frac{u_s}{u_f} - 1\right) \propto -0.63 \log (d_f^3) ,$$

which is in good agreement with (32), but the absolute values of the rates turn out to be much higher than in the above example for a concentration of 2.6%.

Discussing this fact, Kumar et al. [3] point to the possible influence of the mechanism of the physicochemical nature on the interactions of particles with each other. Within the framework of the entrainment theory developed here, this hypothesis can take the following form. Addition of the ions of salt to the solution decreases the forces of repulsion between the particles, thus increasing their tendency toward mutual attraction [14]. This results in the fact that fine particles remain in the field of influence of large ones for a longer time and for a longer time follow them with velocities substantially higher than their intrinsic Stokes values. The role of the forces acting between the particles in sedimentation has been considered in [15] without using specific mechanisms of this influence.

Formula (32) and the experimental data of [2] on sedimentation of fine quartz sand in a plate-type centrifuge have been compared in [4, 5]. A satisfactory confirmation of the inverse-square-law dependence of the sedimentation rate on the size of the particles of the fine fraction and a qualitative agreement regarding the influence of the concentration of the coarse fraction have been found.

NOTATION

d, particle diameter, μm; g, acceleration of gravity or centrifugal acceleration, m/sec^2; r, distance from the center of the cell; r_c and r_f, radius of a coarse particle and a fine particle, m; t, time, sec; $t(y_j) = t_j$, time of stay of the jth particle in the cell, sec; u_c and u_f, Stokes rate of settling of a coarse particle and a fine particle, m/sec; $u(x, y)$ and $v(x, y)$, axial and radial velocities, m/sec; $u_p(y_j)$, mean velocity of traversal of the cell by an individual particle, m/sec; $\langle u_p \rangle$, flux-mean velocity of traversal of the cell by an ensemble of particles, m/sec; y_j, coordinate of the entry of the jth particle into the cell, m; x and y, longitudinal and traverse coordinates, m; C_V, volume fraction of all the particles; C_{V_c}, volume fraction of the coarse fraction; D, dimensionless radius of the cell; R, radius of the cell base, m; Re, Reynolds number; $S(r)$, function; $\langle U \rangle$, dimensionless mean velocity of motion of a particle on a rectilinear portion; U and W, dimensionless axial and radial velocities; X and Y, dimensionless longitudinal and transverse coordinates; Y_0, dimensionless transverse coordinate of the entry of the particle into the cell; α, correction factor; η, dynamic viscosity of the liquid, Pa·sec; λ, coefficient of "neglect" of the particles in the vicinity of the axial line; ν, kinematic viscosity of the liquid, m^2/sec; θ, angle of the spherical coordinate system; ρ, density, kg/m^3; τ, dimensionless time. Subscripts: c, coarse particle; f, fine particle; liq, liquid; p, particle; r, stay (residence); 0, point of shift of the particle to a circular path; 1 and 3, rectilinear and circular portions of the path of a fine particle; *, point of disappearance of the circular portion.

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